KAMPUS BELA NEGARA

M2

OPTIMIZATION PROBLEM FORMULATION AND SEARCH SPACE REPRESENTATION

Objective functions, constraints, solution representation, and search spaces

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Course:
Heuristic Techniques

OUTLINE

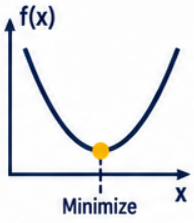
1 INTRODUCTION TO OPTIMIZATION PROBLEM FORMULATION

- Definition of an optimization problem
- From real-world problems to mathematical models
- Identifying the decision-making elements
- Basic optimization model structure



2 OBJECTIVE FUNCTIONS

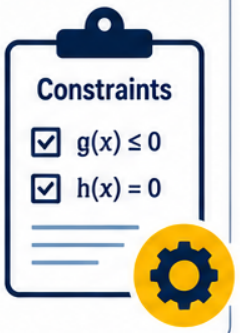
- Definition and purpose of an objective function
- Maximization and minimization
- Single-objective optimization
- Simple examples of objective functions



$$\min f(x)$$

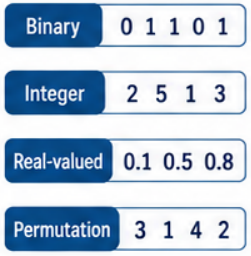
3 CONSTRAINTS

- Definition and role of constraints
- Equality and inequality constraints
- Feasibility conditions
- Examples of resource and operational constraints



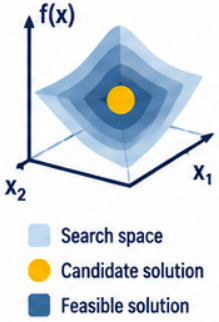
4 SOLUTION REPRESENTATION

- Definition of solution representation
- Common representations: binary, integer, real-valued, and permutation
- Choosing a suitable representation for a problem
- Simple representation examples



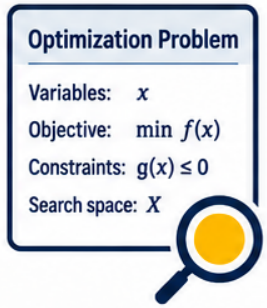
5 SEARCH SPACE REPRESENTATION

- Definition of search space
- Discrete and continuous search spaces
- Candidate solutions within the search space
- Relationship between search space, constraints, and feasible solutions



6 ILLUSTRATIVE PROBLEM & SUMMARY

- Formulating a simple optimization problem
- Identifying variables, objective, constraints, and search space
- Summary of key concepts



1 INTRODUCTION TO OPTIMIZATION PROBLEM FORMULATION

Definition of an optimization problem

What is an Optimization Problem?

An optimization problem is the task of finding the **best solution** from a set of possible solutions, according to a given **objective function**, while satisfying certain **constraints**.

General Formulation

Find x such that

$$\min / \max \quad f(x)$$

subject to $g_i(x) \leq 0 \quad (i = 1, 2, \ldots, m)$

$h_j(x) = 0 \quad (j = 1, 2, \ldots, p)$

$x \in S$

x	Decision variables (vector of variables)
$f(x)$	Objective function (to be minimized or maximized)
$g_i(x)$	Inequality constraints
$h_j(x)$	Equality constraints
S	Feasible solution space

Key Elements

Objective	Variables	Constraints	Solution Space
What to optimize?	What can be decided?	What must be satisfied?	Where to search?

Example (Minimization)

Problem: Minimize production cost

- **Decision variables:** amounts of resources to use
- **Objective:** minimize total cost
- **Constraints:** resource limits, demand requirements, operational rules
- **Goal:** find the lowest cost configuration that satisfies all constraints



1 INTRODUCTION TO OPTIMIZATION PROBLEM FORMULATION

From real-world problems to mathematical models

Why Model Real-World Problems?

Real-world problems are often complex, involve many choices, limited resources, and conflicting objectives. We use mathematical models to **simplify the problem**, **capture the essential elements**, and **find the best solution** using optimization techniques.

From Reality to Mathematical Model

1 Real-World Problem



Identify and understand the real situation, objectives, limitations, and requirements.

2 Formulate Elements



Determine decision variables, objective(s), constraints, and assumptions.

3 Mathematical Model

$$\min / \max \quad f(x)$$
$$\text{subject to} \quad g_i(x) \leq 0$$
$$h_j(x) = 0$$
$$x \in S$$

Translate the problem elements into mathematical equations and expressions.

4 Solve with Optimization Methods



Use optimization algorithms to find the best (optimal) solution.

Example: Production Planning

Real-World Problem

A company wants to produce two products (A and B) to maximize profit, but has limited raw materials, machine time, and labor.

Mathematical Model

$$\max \quad Z = 3x_1 + 5x_2$$
$$\text{subject to} \quad 2x_1 + x_2 \leq 100 \quad \text{(material)}$$
$$x_1 + 2x_2 \leq 80 \quad \text{(machine time)}$$
$$x_1, x_2 \geq 0$$

Key Takeaway

Mathematical models **bridge the gap** between real-world problems and optimization methods, allowing us to **find the best solution** in a systematic and efficient way.

1 INTRODUCTION TO OPTIMIZATION PROBLEM FORMULATION

Identifying the decision-making elements

Key Decision-Making Elements

Formulating an optimization problem requires identifying the main elements involved in decision making, including **decision variables**, **objective function**, **constraints**, and the **solution space**.

1 Decision Variables

Represent the choices we can control.



Example:

$x_1, x_2, \ldots, x_n$

(amounts, locations, schedules, etc.)

2 Objective Function

Defines what we want to optimize (maximize or minimize).



Example:

$\max / \min \ f(x)$

(profit, cost, distance, time, etc.)

3 Constraints

Specify the limitations that must be satisfied.



Example:

$g_i(x) \leq 0, \quad h_j(x) = 0$

(resource limits, requirements, rules, etc.)

4 Solution Space

The set of all possible solutions for the decision variables.



Example:

$x \in S$

(discrete, continuous, or mixed)

Example: Product Planning Problem

Problem Description

A company produces two products (A and B) and wants to determine how many units to produce to maximize profit, considering limited resources.

Identify the Decision-Making Elements

- **Decision variables:** x_1 = units of Product A
  x_2 = units of Product B
- **Objective function:** $\max \ Z = 40x_1 + 30x_2$
- **Constraints:** $2x_1 + x_2 \leq 100$ (material)
  $x_1 + 2x_2 \leq 80$ (labor)
  $x_1, x_2 \geq 0$
- **Solution space:** $x_1, x_2 \geq 0$ (continuous)

Key Takeaway

Identifying the decision-making elements helps us clearly define the optimization problem and build a **correct mathematical model** for finding the best solution.

1 INTRODUCTION TO OPTIMIZATION PROBLEM FORMULATION

Basic optimization model structure

General Structure

Objective function → $\min / \max \quad f(x)$

Constraints →

subject to

$$\begin{cases} g_i(x) \le 0 & (i = 1, 2, \ldots, m) \\ h_j(x) = 0 & (j = 1, 2, \ldots, p) \\ x \in S \end{cases}$$

Decision variables →

where $x = (x_1, x_2, \ldots, x_n)$

Components of the Model

1	**Objective function** $f(x)$	Function to be minimized or maximized (e.g., cost, profit, distance, time).
2	**Constraints** $g_i(x),\ h_j(x)$	Conditions that limit the feasible solutions (e.g., resource limits, physical rules, requirements).
3	**Decision variables** $x_1, x_2, \ldots, x_n$	Variables that represent the decisions to be made (e.g., amounts, locations, schedules).
4	**Solution space** S	Set of all possible values for the decision variables (e.g., continuous, discrete, or mixed).

Example: Production Planning

Objective:
Maximize profit

Decision variables:
- x_1 = units of Product A
- x_2 = units of Product B

$$\max \quad Z = 40x_1 + 30x_2$$

subject to

$$\begin{cases} 2x_1 + x_2 \le 100 & \text{(material)} \\ x_1 + 2x_2 \le 80 & \text{(labor)} \\ x_1, x_2 \ge 0 \end{cases}$$

Key Takeaway

An optimization model combines an **objective function**, **constraints**, **decision variables**, and a **solution space** to represent a real-world problem in a clear and mathematical form.

2 OBJECTIVE FUNCTIONS

Definition and purpose of an objective function

What is an Objective Function?

An objective function is a **mathematical function** that evaluates the quality (value) of a candidate solution based on the decision variables.

$$f(x)$$

x : decision variables

$f(x)$: objective function value

Purpose of an Objective Function

Defines the goal
Specifies what we want to optimize (maximize or minimize).

Guides the search
Provides a measure to compare different candidate solutions.

Reflects real-world objectives
Represents criteria such as cost, profit, distance, time, or efficiency.

Helps find the best solution
The optimization process seeks the solution with the best (optimal) objective function value.

Example

Problem: A company produces two products (A and B) and wants to maximize profit.

Objective function:

$$\max \; Z = 40x_1 + 30x_2$$

Objective function (to be maximized)

Decision variables (amounts of products)

Key Takeaway

The objective function **quantifies what we want to achieve** and serves as the criterion for evaluating and comparing candidate solutions in the optimization process.

2 OBJECTIVE FUNCTIONS

Maximization and minimization

Two Types of Optimization

An objective function can be either **maximized** or **minimized**, depending on the goal of the problem.
Both aim to find the best (optimal) solution from a set of feasible solutions.

Maximization

Find the **largest value** of the objective function.

$$\max \quad f(x)$$

$$x \in S$$

S : feasible solution space

Larger is better

Minimization

Find the **smallest value** of the objective function.

$$\min \quad f(x)$$

$$x \in S$$

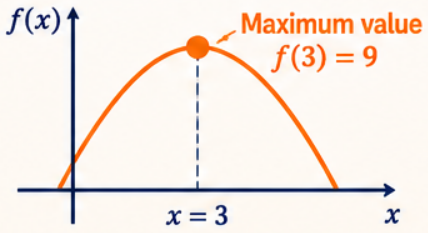
S : feasible solution space

Smaller is better

Illustration on a Simple Function

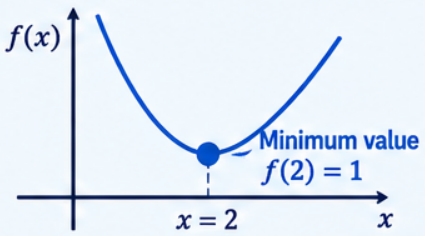
Maximization Example

Find the maximum value of
$f(x) = -(x-3)^2 + 9$



Minimization Example

Find the minimum value of
$f(x) = (x-2)^2 + 1$



Real-World Examples

Maximization Problems

- Maximize profit
- Maximize production
- Maximize efficiency
- Maximize customer satisfaction

Goal: Achieve the largest possible value.

Minimization Problems

- Minimize cost
- Minimize time
- Minimize distance
- Minimize error or risk

Goal: Achieve the smallest possible value.

2 OBJECTIVE FUNCTIONS

Single-objective optimization

What is Single-Objective Optimization?

Single-objective optimization focuses on optimizing **one objective function**, either to maximize or minimize, subject to a set of constraints.
The goal is to find the best feasible solution with respect to a single performance measure.

Mathematical Formulation

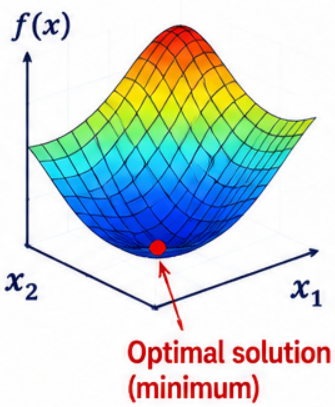
$$\min / \max \quad f(x)$$

subject to

$$\begin{cases} g_i(x) \le 0 & (i = 1, 2, \ldots, m) \\ h_j(x) = 0 & (j = 1, 2, \ldots, p) \\ x \in S \end{cases}$$

$f(x)$: objective function (single objective)
x : decision variables
$g_i(x)$: inequality constraints
$h_j(x)$: equality constraints
S : feasible solution space

Search for the Optimal Solution



Single-objective optimization aims to find the best point (minimum or maximum) in the feasible region that gives the optimal value of the objective function.

Example: Minimization Problem

Find the minimum value of

$$f(x) = x_1^2 + x_2^2$$

subject to

$$x_1 + x_2 \ge 1$$
$$x_1, x_2 \ge 0$$

Optimal solution:

$$x^* = (0.5, 0.5)$$
$$f(x^*) = 0.5$$

Key Characteristics

- Only one objective function to optimize
- Can be a maximization or minimization problem
- Subject to constraints
- Has a single optimal solution (or a set of optimal solutions)
- Widely used in real-world optimization problems

Key Takeaway

Single-objective optimization provides a clear and focused goal, allowing us to find the best feasible solution with respect to **one performance measure**.

2 OBJECTIVE FUNCTIONS

Simple examples of objective functions

An objective function can take many forms depending on the problem. Here are some simple and commonly used examples.

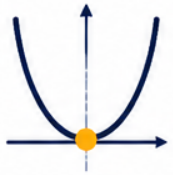
1 Linear Objective Function

$$\max \quad Z = 3x_1 + 2x_2$$

A linear function that maximizes a weighted sum of the decision variables.

2 Quadratic Objective Function

$$\min \quad Z = x_1^2 + x_2^2$$



A quadratic function that minimizes the sum of squared values of the decision variables.

3 Absolute Value Objective Function

$$\min \quad Z = |x_1| + |x_2|$$



An objective function that minimizes the sum of absolute values, often used for sparse solutions.

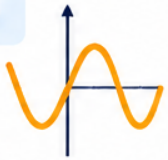
4 Weighted Objective Function

$$\max \quad Z = w_1x_1 + w_2x_2 + w_3x_3$$

A linear function with different weights, allowing different importance for each decision variable.

5 Nonlinear Objective Function

$$\min \quad Z = \sin(x_1) + x_2^2$$



A nonlinear function that can model more complex relationships between variables.

6 Real-World Example

$$\max \quad Z = 50x_1 + 40x_2$$



Maximize profit, where x_1 and x_2 are the numbers of two products produced.

3 CONSTRAINTS

Definition and role of constraints

What are Constraints?

Constraints are **conditions or restrictions** that limit the set of feasible solutions in an optimization problem.

They specify what solutions are allowed or not allowed.

Mathematical Form

Constraints are usually expressed as equality or inequality relations:

$$g_i(x) \leq 0 \quad (i = 1, 2, \ldots, m)$$
$$h_j(x) = 0 \quad (j = 1, 2, \ldots, p)$$
$$x \in S$$

$g_i(x)$: inequality constraints
$h_j(x)$: equality constraints
x : decision variables
S : feasible solution space

Role of Constraints

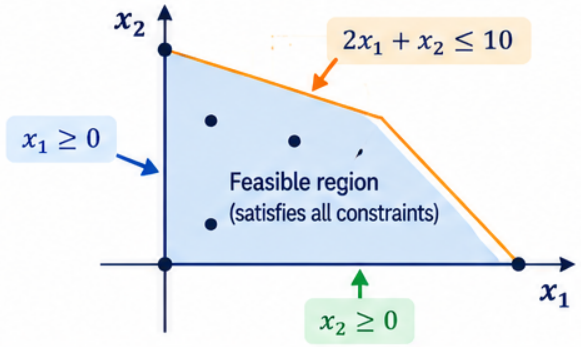
Limit the feasible solutions
Reduce the search space to only valid solutions.

Represent real-world restrictions
Model practical limitations such as resources, capacity, policies, or physical laws.

Ensure realistic and implementable solutions
Help find solutions that can actually be applied in practice.

Work together with the objective function
The objective function defines what we want to optimize, while constraints define what we are allowed to do.

Illustration



Infeasible vs. Feasible Solutions

- Feasible solution (satisfies all constraints)
- Infeasible solution (violates at least one constraint)

Only solutions inside the feasible region can be considered by the optimization algorithm.

Real-World Examples

Production Planning
- Limited resources (e.g., labor, materials)
- Production capacity

Transportation
- Vehicle capacity
- Time windows
- Route limitations

Budget Allocation
- Total budget limit
- Spending constraints for each category

3 CONSTRAINTS

Equality and inequality constraints

Equality Constraints

An equality constraint requires that a relationship **holds exactly** (i.e., the left-hand side equals the right-hand side).

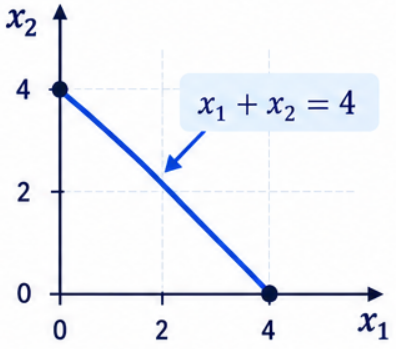
$$h(x) = 0$$

e.g. $x_1 + x_2 = 4$

Example

The solutions must satisfy $x_1 + x_2 = 4$.

The feasible solutions lie on a straight line.



Inequality Constraints

An inequality constraint restricts the solutions to be **less than or equal to (≤)** or **greater than or equal to (≥)** a value.

$$g(x) \leq 0 \quad \text{or} \quad g(x) \geq 0$$

e.g. $2x_1 + x_2 \leq 6$

$x_1 \geq 0, \quad x_2 \geq 0$

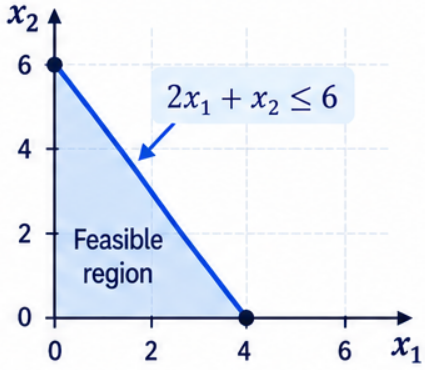
Example

The solutions must satisfy

$2x_1 + x_2 \leq 6,$

$x_1 \geq 0, \quad x_2 \geq 0.$

The feasible solutions form a shaded region (a half-plane).



Key Takeaway

Equality constraints define a lower-dimensional set of solutions (e.g., a **line or surface**), while inequality constraints define a **region of feasible solutions**.

3 CONSTRAINTS

Feasibility conditions

What Are Feasibility Conditions?

Feasibility conditions specify the conditions that a solution must satisfy to be considered feasible, i.e., to meet all the constraints of the problem.

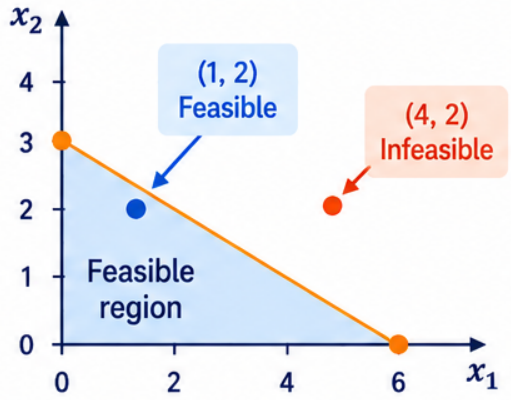
A feasible solution:

- Satisfies all equality constraints
- Satisfies all inequality constraints
- Belongs to the feasible solution space

Feasible vs. Infeasible Solutions

Consider the constraints:

$$x_1 + 2x_2 \leq 6$$
$$x_1 \geq 0, \quad x_2 \geq 0$$



Examples of Feasibility Conditions

Example 1

A solution is feasible if it satisfies:

$$x_1 + x_2 \leq 10$$
$$x_1 - x_2 = 2$$
$$x_1 \geq 0, \quad x_2 \geq 0$$

The point (6, 4) is feasible because:

- $6 + 4 \leq 10$ ✓
- $6 - 4 = 2$ ✓
- $6 \geq 0, \quad 4 \geq 0$ ✓

Example 2

A solution is infeasible if it violates any constraint.

$$x_1 + 2x_2 \leq 6$$
$$x_1 \geq 0, \quad x_2 \geq 0$$

The point (5, 2) is infeasible because:

- ✗ $5 + 2(2) = 9 > 6$ (violates the first constraint)

Key Takeaway

A solution is **feasible** if and only if it satisfies **all the constraints** (equality and inequality). The set of all feasible solutions forms the **feasible region** (feasible solution space).

3 CONSTRAINTS

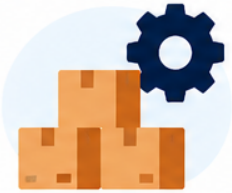
Examples of resource and operational constraints

Resource Constraints

Resource constraints **limit the amount of available resources** that can be used in the optimization problem, such as budget, raw materials, labor, or machine time.

Example: Production Planning

A factory produces two products, A and B. Each unit of A requires 2 kg of raw material, and each unit of B requires 3 kg. The total available raw material is 600 kg.



Resource constraint (raw material):

$$2x_1 + 3x_2 \leq 600$$

x_1 : number of units of product A
x_2 : number of units of product B

The total amount of raw material used **cannot exceed** the available 600 kg.

Operational Constraints

Operational constraints represent **practical rules or requirements** in the operation of a system, such as capacity limits, demand requirements, or logical relationships between decision variables.

Example: Machine Capacity

A company produces two products, A and B. Each unit of A requires 1 hour of machine time, and each unit of B requires 2 hours. The machine is available for at most 400 hours per week.



Operational constraint (machine time):

$$x_1 + 2x_2 \leq 400$$

x_1 : number of units of product A
x_2 : number of units of product B

The total machine time used **cannot exceed** the available 400 hours per week.

Key Takeaway

Resource constraints ensure that solutions stay within **available resources**, while operational constraints enforce **practical rules and limitations** in the system.

4 SOLUTION REPRESENTATION

Definition of solution representation

What is Solution Representation?

Solution representation is the way a solution to an optimization problem is **encoded or described** so that it can be **processed**, **evaluated**, **and manipulated** by an algorithm.

It defines the structure and format of candidate solutions in the search space.

Key Points

Encoding
Represents decision variables in a specific format (e.g., vector, string, matrix).

Evaluation
Allows the objective function and constraints to be computed from the representation.

Manipulation
Enables operators (e.g., mutation, crossover, neighborhood moves) to modify solutions.

Problem-dependent
The representation depends on the type of optimization problem and the algorithm used.

Example

For a problem with two decision variables (x_1, x_2), a solution can be represented as:

Vector representation:

$$\boldsymbol{x} = (x_1, x_2) = (3, 5)$$

This vector encodes a candidate solution where $x_1 = 3$ and $x_2 = 5$.

Other Representations

- Binary string (e.g., 0101)
- Integer vector (e.g., [2, 4, 1])
- Real-valued vector (e.g., [3.5, 1.2])
- Permutation (e.g., [1, 3, 2, 4])

A good solution representation should be **simple, effective, and suitable for the problem and algorithm.**

4 SOLUTION REPRESENTATION

Common representations: binary, integer, real-valued, and permutation

01 Binary Representation

A solution is represented by a string of **binary digits (0 or 1).**

Example

$$x = [1, 0, 1, 1, 0]$$

A 5-bit string representing a candidate solution.

Common Use Cases

- Feature selection
- On/off decision variables
- Combinatorial optimization

123 Integer Representation

A solution is represented by a vector of **integer values** (positive, negative, or zero).

Example

$$x = [3, -1, 0, 7]$$

A 4-dimensional integer vector representing a candidate solution.

Common Use Cases

- Resource allocation
- Scheduling
- Discrete parameter optimization

Real-valued Representation

A solution is represented by a vector of **real (continuous) values.**

Example

$$x = [0.25, -1.6, 3.8]$$

A 3-dimensional real-valued vector representing a candidate solution.

Common Use Cases

- Parameter tuning
- Continuous optimization
- Neural network optimization

Permutation Representation

A solution is represented by an **ordered arrangement (permutation)** of a set of elements.

Example

$$x = [4, 2, 1, 3]$$

A permutation of {1, 2, 3, 4} representing a candidate solution (e.g., visiting order).

Common Use Cases

- Traveling Salesman Problem (TSP)
- Task sequencing
- Ordering and routing problems

Key Takeaway

The choice of solution representation depends on the **problem type** and directly affects how solutions are **generated, evaluated,** and **manipulated** by the optimization algorithm.

4 SOLUTION REPRESENTATION

Choosing a suitable representation for a problem

Why It Matters?

The choice of solution representation affects how solutions are **generated**, **evaluated**, **and manipulated** by the optimization algorithm. A suitable representation can make the search **more efficient and effective**.

A good representation matches the nature of the problem and the algorithm used.

Key Considerations

Problem Nature
Discrete or continuous variables? Fixed order or unordered set?

Variable Characteristics
Binary, integer, real-valued, or permutation?

Constraints
Does the representation easily handle problem constraints?

Algorithm Compatibility
Is the representation suitable for the chosen optimization algorithm (e.g., GA, SA, PSO)?

Computational Efficiency
Does it allow efficient encoding, decoding, and solution evaluation?

Guidelines and Examples

Problem Type	Suitable Representation	Example
Selection / Yes–No decision	Binary	Feature selection $x = [1, 0, 1, 0]$
Counting / Discrete quantities	Integer	Number of machines $x = [3, 5, 2]$
Continuous parameter optimization	Real-valued	Parameter tuning $x = [0.5, -1.2, 3.0]$
Ordering / Sequencing	Permutation	Job scheduling $x = [4, 2, 1, 3]$

There is no single best representation. Choose the one that best fits the **problem structure**, **constraints**, and **algorithm**.

Key Takeaway

A suitable solution representation **simplifies the search process**, **respects the problem constraints**, and **improves the performance** of the optimization algorithm.

4 SOLUTION REPRESENTATION

Simple representation examples

01 Binary Representation

A solution is represented by a string of **binary digits (0 or 1)**.

Example

$$x = [1, 0, 1, 1, 0]$$

Can represent a selection decision (e.g., whether to include an item).

Example problem:
Feature selection

1	0	1	1	0
Include	Exclude	Include	Include	Exclude

02 Integer Representation

A solution is represented by a vector of **integer values** (positive, negative, or zero).

Example

$$x = [3, -1, 0, 5]$$

Can represent counts or discrete quantities.

Example problem:
Machine allocation



3	0	2	5
Machine 1 (units)	Machine 2 (units)	Machine 3 (units)	Machine 4 (units)

03 Real-valued Representation

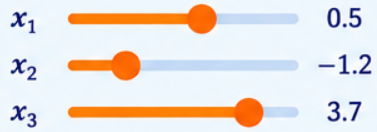
A solution is represented by a vector of **real (continuous) values**.

Example

$$x = [0.5, -1.2, 3.7]$$

Can represent continuous decision variables (e.g., parameters).

Example problem:
Parameter optimization



04 Permutation Representation

A solution is represented by an **ordered arrangement (permutation)** of elements.

Example

$$x = [4, 2, 1, 3]$$

Can represent a sequence or ordering of items.

Example problem:
Traveling Salesman Problem (TSP)



Visit cities in the order:
1 → 2 → 3 → 4

Key Takeaway

Different representation schemes are used depending on the **problem type**.
Each representation encodes a candidate solution in a **suitable and meaningful way**.

5 SEARCH SPACE REPRESENTATION

Definition of search space

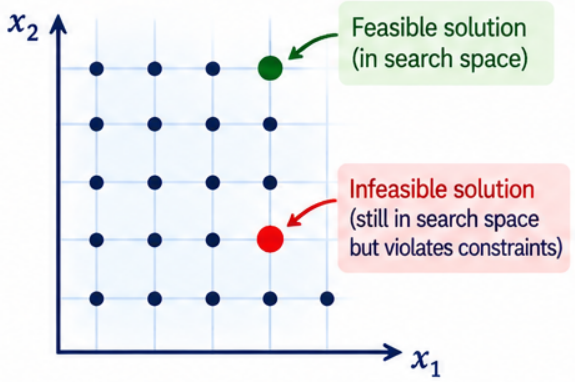
What is Search Space?

The search space is the set of **all possible candidate solutions** that can be considered in an optimization problem.

Each point in the search space represents a solution (a specific assignment of decision variables), which may be **feasible** or **infeasible**, and has an associated objective function value.

Illustration

For a problem with two decision variables (x_1, x_2), the search space can be visualized as a two-dimensional plane.



Each dot represents a candidate solution.

Key Points

1. **Set of All Possible Solutions**
   The search space contains every candidate solution that can be represented by the chosen solution representation.
2. **Depends on the Representation**
   Its size and structure depend on the type of variables (binary, integer, real-valued, permutation, etc.).
3. **Includes Feasible and Infeasible Solutions**
   The search space may contain solutions that violate constraints.
4. **Can Be Very Large**
   For many real-world problems, the search space is enormous, making intelligent search strategies essential.

Key Takeaway

The search space is the set of **all candidate solutions**.
Understanding its structure helps in designing effective search algorithms.

5 SEARCH SPACE REPRESENTATION

Discrete and continuous search spaces

Discrete Search Space

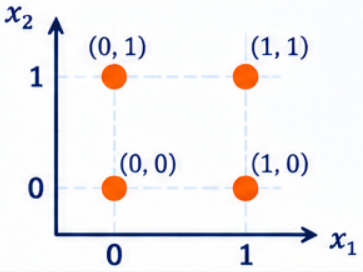
A discrete search space consists of a **countable (often finite) set** of candidate solutions. The variables can take only **specific, separate values**.

Example

A problem with two binary variables (x_1, x_2) has four possible solutions.

Search space:
$S = \{(0,0), (0,1), (1,0), (1,1)\}$

Search space visualization (binary variables)



Typical applications:
combinatorial optimization, scheduling, feature selection, assignment problems.

Continuous Search Space

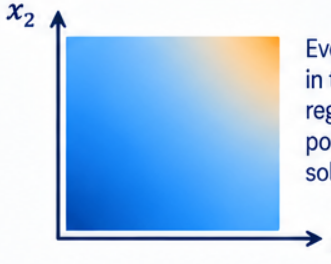
A continuous search space consists of an **uncountably infinite set** of candidate solutions. The variables can take **any real value** within a given range.

Example

A problem with two real-valued variables (x_1, x_2) has infinitely many possible solutions.

Search space:
$S = \{(x_1, x_2) \mid x_1, x_2 \in \mathbb{R}\}$

Search space visualization (real-valued variables)



Typical applications:
parameter optimization, machine learning, engineering design, control problems.

Key Takeaway

The search space can be **discrete** or **continuous**, depending on the type of variables and the problem. Understanding the search space helps in **choosing suitable search algorithms**.

5 SEARCH SPACE REPRESENTATION

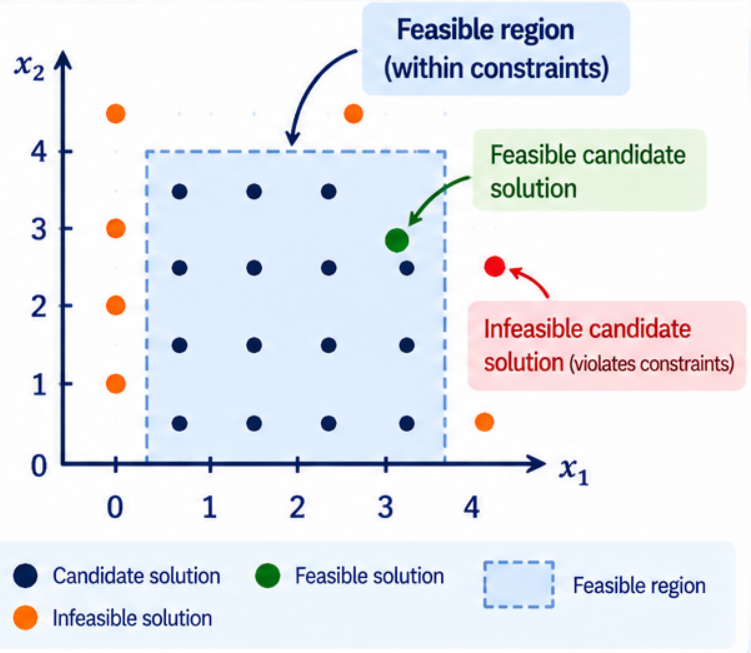
Candidate solutions within the search space

Definition

A candidate solution is a **specific point in the search space** that represents a possible solution to the optimization problem.

- Each candidate solution is defined by a set of values for the decision variables.
- The search space contains all candidate solutions.
- Some candidates are **feasible** (satisfy the constraints), while others are **infeasible**.

Illustration



Example Candidates

Example 1: Discrete search space (binary variables)

Candidate solution: $x = [1, 0, 1, 0]$

This represents a specific point in the search space (one possible solution).

Example 2: Continuous search space (real-valued variables)

Candidate solution: $x = [2.5, 1, 8]$

This is one candidate solution within a continuous search space.

Example 3: Permutation search space

Candidate solution: $x = [3, 1, 4, 2]$

This represents one possible ordering (permutation) in the search space.

Key Takeaway

A candidate solution is a **point in the search space**. The search space contains all possible candidates, and the goal is to find the **best feasible candidate** according to the objective function.

5 SEARCH SPACE REPRESENTATION

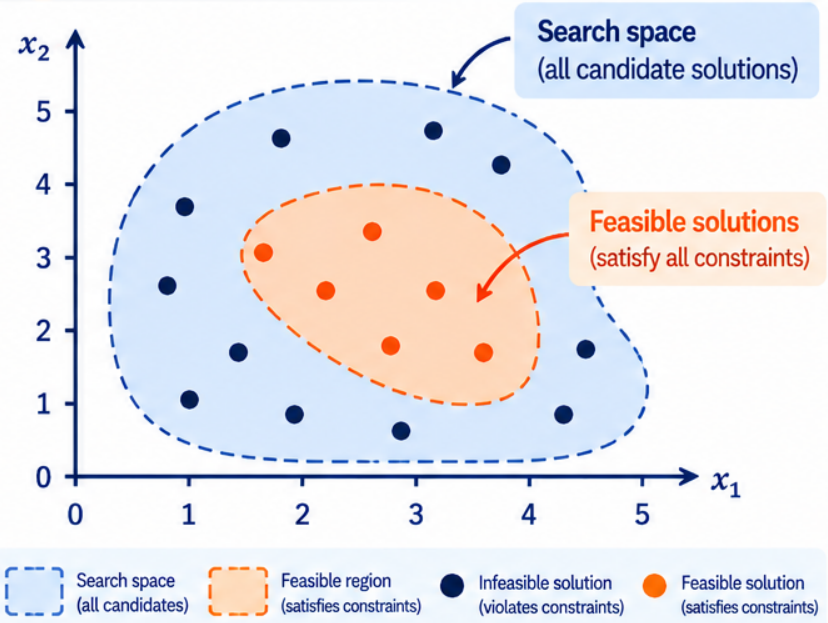
Relationship between search space, constraints, and feasible solutions

Concept

The search space contains **all possible candidate solutions**. **Constraints** restrict this space, and the **feasible solutions** are the candidates that satisfy all constraints.

The optimization algorithm searches within the feasible region to find the best solution.

Illustration



Key Points

1. **Search Space**
   The set of **all possible** candidate **solutions** for the problem.
2. **Constraints**
   Conditions that **restrict** which candidates are allowed (e.g., resource limits, logical rules, physical limitations).
3. **Feasible Solutions**
   Candidates that **satisfy every constraint**. They form the feasible region, which is a subset of the search space.

Key Takeaway

Constraints **reduce the search space** to a **feasible region**, and the optimization algorithm searches within this region to find the **best solution**.

6 ILLUSTRATIVE PROBLEM & SUMMARY

Formulating a simple optimization problem

Steps in Formulating an Optimization Problem

1. **Define decision variables**
   Identify what needs to be decided (e.g., quantities, selections).
2. **Specify the objective function**
   Define what to optimize (maximize or minimize).
3. **Set constraints**
   Identify the conditions that must be satisfied.
4. **Determine the search space**
   Define the set of all possible candidate solutions.

Example: Production Planning Problem

A small factory produces two products, A and B.
Determine how many units of each product to produce to maximize profit subject to resource constraints.

Problem Formulation

Decision variables:
x_1 = number of units of product A
x_2 = number of units of product B

Objective function (maximize profit):
Maximize $Z = 40x_1 + 30x_2$

Constraints (resource limits):

$2x_1 + x_2 \leq 100$ (Labor hours)
$x_1 + 2x_2 \leq 80$ (Machine hours)
$x_1, x_2 \geq 0$ (Non-negativity)

Interpretation

- x_1 and x_2 are the decision variables (search space: all non-negative values).
- The objective function represents the total profit.
- The constraints limit the use of labor and machine hours.
- The goal is to find the values of x_1 and x_2 within the feasible region that give the maximum profit.

Key Takeaway

Formulating an optimization problem involves defining **decision variables**, an **objective function, constraints**, and the **search space**. A clear formulation helps in designing and applying the right optimization algorithm.

6 ILLUSTRATIVE PROBLEM & SUMMARY

Example: Identifying variables, objective, constraints, and search space

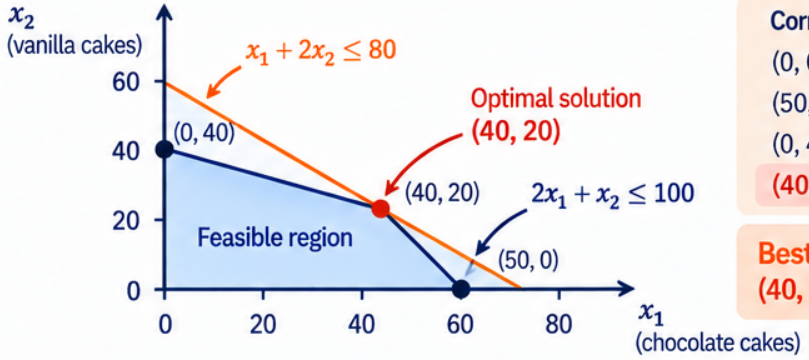
Problem Statement



A bakery produces two types of cakes: chocolate (C) and vanilla (V).
Each chocolate cake requires 2 kg of flour and 1 hour of baking time.
Each vanilla cake requires 1 kg of flour and 2 hours of baking time.
The bakery has at most 100 kg of flour and 80 hours of baking time per week. The profit is $5 per chocolate cake and $4 per vanilla cake.

How many cakes of each type should be produced to maximize the total profit?

Graphical Illustration (Feasible Region)



Corner Points

(0, 0)	$Z = 0$
(50, 0)	$Z = 250$
(0, 40)	$Z = 160$
(40, 20)	$Z = 280$

Best solution:
(40, 20) with profit 280

Solution

1 Decision Variables

Let x_1 = number of chocolate cakes (C)
Let x_2 = number of vanilla cakes (V)

2 Objective Function

Maximize $Z = 5x_1 + 4x_2$
(total profit in dollars)

3 Constraints

$2x_1 + x_2 \leq 100$ (flour constraint)
$x_1 + 2x_2 \leq 80$ (baking time constraint)
$x_1, x_2 \geq 0$ (non-negativity)

4 Search Space

$$S = \{(x_1, x_2) \in \mathbb{R}^2 \mid 2x_1 + x_2 \leq 100,\ x_1 + 2x_2 \leq 80,\ x_1 \geq 0,\ x_2 \geq 0\}$$

5 Optimal Solution

$x_1 = 40$ (chocolate cakes)
$x_2 = 20$ (vanilla cakes)
Maximum profit: $Z = 5(40) + 4(20) = 280$

Key Takeaway

By identifying the **decision variables**, **objective function**, **constraints**, and **search space**, we can formulate an optimization problem and find the **best feasible solution**.

6 ILLUSTRATIVE PROBLEM & SUMMARY

Identifying variables, objective, constraints, and search space

Problem Scenario

A small factory produces two products, A and B.
Each product requires labor hours and machine hours.
The factory has limited resources and wants to **maximize the total profit**.



Goal:
Find the best production plan for products A and B.

1 Variables

Let:

x_1 = number of units of product A

x_2 = number of units of product B

Decision variables represent the quantities to be determined.

2 Objective Function

Maximize the total profit:

$$Z = 40x_1 + 30x_2$$

where 40 and 30 are the profit per unit for products A and B, respectively.

3 Constraints

Subject to resource limits:

$$2x_1 + x_2 \leq 100$$ (Labor hours)

$$x_1 + 2x_2 \leq 80$$ (Machine hours)

$$x_1, x_2 \geq 0$$ (Non-negativity)

Constraints define the feasible solutions.

4 Search Space

The search space is the set of all possible values of the decision variables:

$$S = \{(x_1, x_2) \mid x_1 \geq 0, x_2 \geq 0\}$$

The **feasible region** is the subset of S that satisfies all constraints:

$$F = \{(x_1, x_2) \in S \mid 2x_1 + x_2 \leq 100,\ x_1 + 2x_2 \leq 80\}$$

Key Takeaway

Formulating an optimization problem involves identifying the **decision variables**, **objective function**, **constraints**, and **search space**, which together define the feasible solutions for finding the best solution.

6 ILLUSTRATIVE PROBLEM & SUMMARY

Summary of key concepts

1 Solution Representation

How a candidate solution is encoded.

Common representations: binary, integer, real-valued, and permutation.

2 Search Space

The set of all possible candidate solutions.

Can be **discrete** (finite or countable) or continuous (infinite).

3 Constraints

Conditions that restrict which solutions are allowed.

The **feasible region** contains all solutions that satisfy all constraints.

4 Objective Function

A function that evaluates the quality of a solution.

Can be a **maximization** or **minimization** problem.

5 Feasible Solution

A candidate solution that satisfies all constraints.

The goal is to find the best **feasible solution** according to the objective function.

6 Optimization Problem

Find the best solution (from the feasible region) that optimizes the objective function.

Brings together representation, search space, constraints, and objective.

Key Takeaway

Formulating an optimization problem involves defining the **solution representation**, **search space**, **constraints**, and **objective function** to find the best feasible solution.

TASK

1 DOWNLOAD A RESEARCH ARTICLE

Download a scientific journal article (published **no later than 2020**) that applies a heuristic optimization technique to solve a real-world problem. Read and understand the article.



2 IDENTIFY AND EXPLAIN THE PROBLEM FORMULATION

From the article, identify and explain the optimization problem, including:

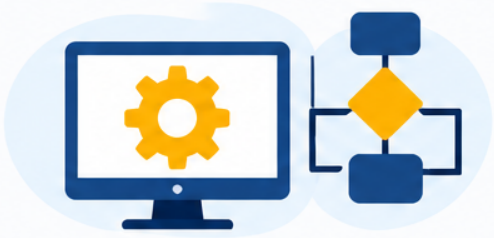
- **Decision variables** – what are the variables to be optimized?
- **Objective function** – what is the objective (minimize or maximize)?
- **Constraints** – what are the constraints (if any)?
- **Search space** – how is the search space defined or represented?

$$\min / \max \quad f(x)$$
$$\text{s.t.}$$
$$g_i(x) \leq 0 \quad (i = 1, \ldots, m)$$
$$h_j(x) = 0 \quad (j = 1, \ldots, p)$$
$$x \in S$$

3 ANALYZE THE HEURISTIC METHOD

Explain the heuristic optimization method used in the article, including:

- Main idea of the method
- How the method is applied to the problem
- Key parameters (if any)
- Summary of the main results and performance comparison (if available)



4 WRITE A BRIEF REVIEW

Prepare a concise review in **one page** that covers the points above. Use your own words and include a proper reference to the article.

