MATERI 11. VEKTOR

- Ruang Berdimensi n Euclidean
- Transformasi Linier R^n ke R^m

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Ruang-n Euclidean

(Euclidean n-space)

Review: *Bab 3 membahas Ruang-2 dan Ruang-3*

Ruang-n : himpunan yang beranggotakan vektor-vektor dengan n komponen

$$\{ \ldots , \mathbf{v} = (v_1, v_2, v_3, v_4, \ldots, v_n), \ldots.. \}$$

- Atribut: arah dan "panjang" / norma $||\mathbf{v}||$
- Aritmatika vektor-vektor di Ruang-n:
    1. Penambahan vektor
    2. Perkalian vektor dengan skalar
    3. Perkalian vektor dengan vektor

Norma sebuah vektor:

Norma Euclidean (*Euclidean norm*) di Ruang-n :

$$\mathbf{u} = (u_1, u_2, u_3, \ \dots \ , u_n)$$

$$\boxed{\|\mathbf{u}\| = \sqrt{u_1^2 + u_2^2 + u_3^2 + \ \dots + u_n^2}}$$

Penambahan vektor: di Ruang-n

$\mathbf{u} = (u_1, u_2, u_3, \ldots, u_n)$; $\mathbf{v} = (v_1, v_2, v_3, \ldots, v_n)$

$\mathbf{w} = (w_1, w_2, w_3, \ldots, w_n) = \mathbf{u} + \mathbf{v}$

$\mathbf{w} = (u_1, u_2, u_3, \ldots, u_n) + (v_1, v_2, v_3, \ldots, v_n)$

$$\boxed{\mathbf{w} = (u_1 + v_1, u_2 + v_2, u_3 + v_3, \ldots, u_n + v_n)}$$

$w_1 = u_1 + v_1$

$w_2 = u_2 + v_2$

…………

$w_2 = u_n + v_n$

Negasi suatu vektor:

$\mathbf{u} = (u_1, u_2, u_3, \ldots, u_n)$

$-\mathbf{u} = (-u_1, -u_2, -u_3, \ldots, -u_n)$

Selisih dua vektor:

$$\mathbf{w} = \mathbf{u} - \mathbf{v} = \mathbf{u} + (-\mathbf{v})$$

$$= (u_1 - v_1, u_2 - v_2, u_3 - v_3, \ldots, u_n - v_n)$$

Vektor nol: $\mathbf{0} = (0_1, 0_2, 0_3, \ldots, 0_n)$

Perkalian skalar dengan vektor:

$$\boxed{\mathbf{w} = k\mathbf{v} = (k\mathrm{v}_1, k\mathrm{v}_2, k\mathrm{v}_3, \ldots, k\mathrm{v}_n)}$$

$$(\mathrm{w}_1, \mathrm{w}_2, \mathrm{w}_3, \ldots, \mathrm{w}_n) = (k\mathrm{v}_1, k\mathrm{v}_2, k\mathrm{v}_3, \ldots, k\mathrm{v}_n)$$

$$\mathrm{w}_1 = k\mathrm{v}_1$$

$$\mathrm{w}_2 = k\mathrm{v}_2$$

$$\ldots\ldots\ldots$$

$$\mathrm{w}_n = k\mathrm{v}_n$$

Perkalian titik: (perkalian Euclidean)

$$\mathbf{u} \cdot \mathbf{v} = \text{skalar}$$

$$\boxed{\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2 + u_3v_3 + \ldots + u_nv_n}$$

$\mathbf{u} \cdot \mathbf{v} = 0$ jika **u** dan **v** ortogonal

Catatan: perkalian silang hanya di Ruang-3

Aritmatika vektor di Ruang-n:

Teorema 4.1.1.: u, v, w vektor-vektor di Ruang-n

k, l adalah skalar (bilangan *real*)

- u + v = v + u
- (u + v) + w = u + (v + w)
- u + 0 = 0 + u = u
- u + (-u) = (-u) + u = 0
- k(lu) = (kl)u
- k(u + v) = ku + kv
- (k + l) u = ku + lu
- 1u = u

Teorema 4.1.2:

Vektor-vektor **u**, **v**, **w** di Ruang-n; ***k*** adalah skalar

- $\mathbf{u} \,.\, \mathbf{v} = \mathbf{v} \,.\, \mathbf{u}$
- $\mathbf{u} \,.\, (\mathbf{v} + \mathbf{w}) = \mathbf{u} \,.\mathbf{v} + \mathbf{u} \,.\mathbf{w}$
- $k(\mathbf{u} \,.\, \mathbf{v}) = (k\mathbf{u}) \,.\, \mathbf{v} = \mathbf{u} \,.\, (k\mathbf{v})$
- $\mathbf{v} \,.\mathbf{v} > 0$ jika $\mathbf{v} \neq \mathbf{0}$

  $\mathbf{v} \,.\, \mathbf{v} = 0$ jika dan hanya jika $\mathbf{v} = \mathbf{0}$

Teorema 4.1.3 - 4.1.5:

$| \mathbf{u} \cdot \mathbf{v} | \leq \| \mathbf{u} \| \| \mathbf{v} \|$

$\| \mathbf{u} \| \geq 0$

$\| \mathbf{u} \| \geq 0$ jika dan hanya jika $\mathbf{u} = \mathbf{0}$

$\| k\mathbf{u} \| = | k | \| \mathbf{u} \|$

$\| \mathbf{u} + \mathbf{v} \| \leq \| \mathbf{u} \| + \| \mathbf{v} \|$

$d(\mathbf{u}, \mathbf{v}) \geq 0$

$d(\mathbf{u}, \mathbf{v}) = 0$ jika dan hanya jika $\mathbf{u} = \mathbf{v}$

$d(\mathbf{u}, \mathbf{v}) = d(\mathbf{v}, \mathbf{u})$

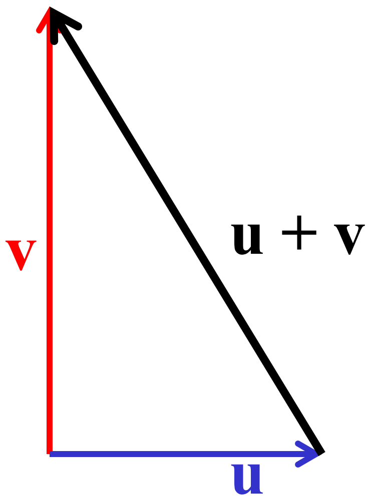
$d(\mathbf{u}, \mathbf{v}) \leq d(\mathbf{u}, \mathbf{w}) + d(\mathbf{w}, \mathbf{v})$

Teorema 4.1.6 – 4.1.7:

$$\mathbf{u} \cdot \mathbf{v} = \tfrac{1}{4} \| \mathbf{u} + \mathbf{v} \|^2 - \tfrac{1}{4} \| \mathbf{u} - \mathbf{v} \|^2$$

Teorema Pythagoras

$$\| \mathbf{u} + \mathbf{v} \|^2 = \| \mathbf{u} \|^2 + \| \mathbf{v} \|^2$$



Perkalian Titik (***dot product***) dikerjakan dengan

perkalian matriks

$$\mathbf{u} = (u_1, u_2, u_3, \ldots, u_n); \quad \mathbf{v} = (v_1, v_2, v_3, \ldots, v_n)$$

$$\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2 + u_3 v_3 + \ldots + u_n v_n$$

Kalau vektor u dan vektor v masing-masing ditulis dalam notasi matriks <u>baris</u>, maka

$$\mathbf{u} \cdot \mathbf{v} = (u_1 \;\; u_2 \;\; u_3 \;\; \ldots \;\; u_n) \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_n \end{pmatrix}$$

$$\boxed{\mathbf{u} \cdot \mathbf{v} = (\mathbf{u})\,(\mathbf{v})^T}$$

$\mathbf{u} = (u_1, u_2 , u_3, \ldots, u_n); \quad \mathbf{v} = (v_1, v_2 , v_3, \ldots, v_n)$

$\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2 + u_3v_3 + \ldots + u_nv_n$

Kalau vektor u dan vektor v masing-masing ditulis dalam notasi matriks <u>kolom</u>, maka

$$\mathbf{u} \cdot \mathbf{v} = \begin{pmatrix} u_1 & u_2 & u_3 & \ldots & u_n \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_n \end{pmatrix} = \mathbf{v} \cdot \mathbf{u}$$

$$\boxed{\begin{aligned} \mathbf{u} \cdot \mathbf{v} &= (\mathbf{u})^T (\mathbf{v}) \\ \mathbf{v} \cdot \mathbf{u} &= (\mathbf{v})^T (\mathbf{u}) \\ \mathbf{u} \cdot \mathbf{v} &= (\mathbf{v})^T (\mathbf{u}) \end{aligned}}$$

Matriks A (n x n), **u** dan **v** masing-masing vektor kolom

$$A\mathbf{u} \cdot \mathbf{v} = (A\mathbf{u}) \cdot \mathbf{v} \overset{?}{=} \mathbf{u} \cdot (A^T\mathbf{v})$$

$$\mathbf{u} \cdot A\mathbf{v} \overset{?}{=} \mathbf{A^T u} \cdot \mathbf{v}$$

$$\begin{aligned}(A\mathbf{u}) \cdot \mathbf{v} &= \mathbf{v} \cdot (A\mathbf{u}) \\ &= \mathbf{v}^T(A\mathbf{u}) \\ &= (\mathbf{v}^T A)\mathbf{u} \\ &= (A^T\mathbf{v})^T\, \mathbf{u} \\ &= \mathbf{u} \cdot (A^T\mathbf{v})\end{aligned}$$